STANTEC

SYSTEMS

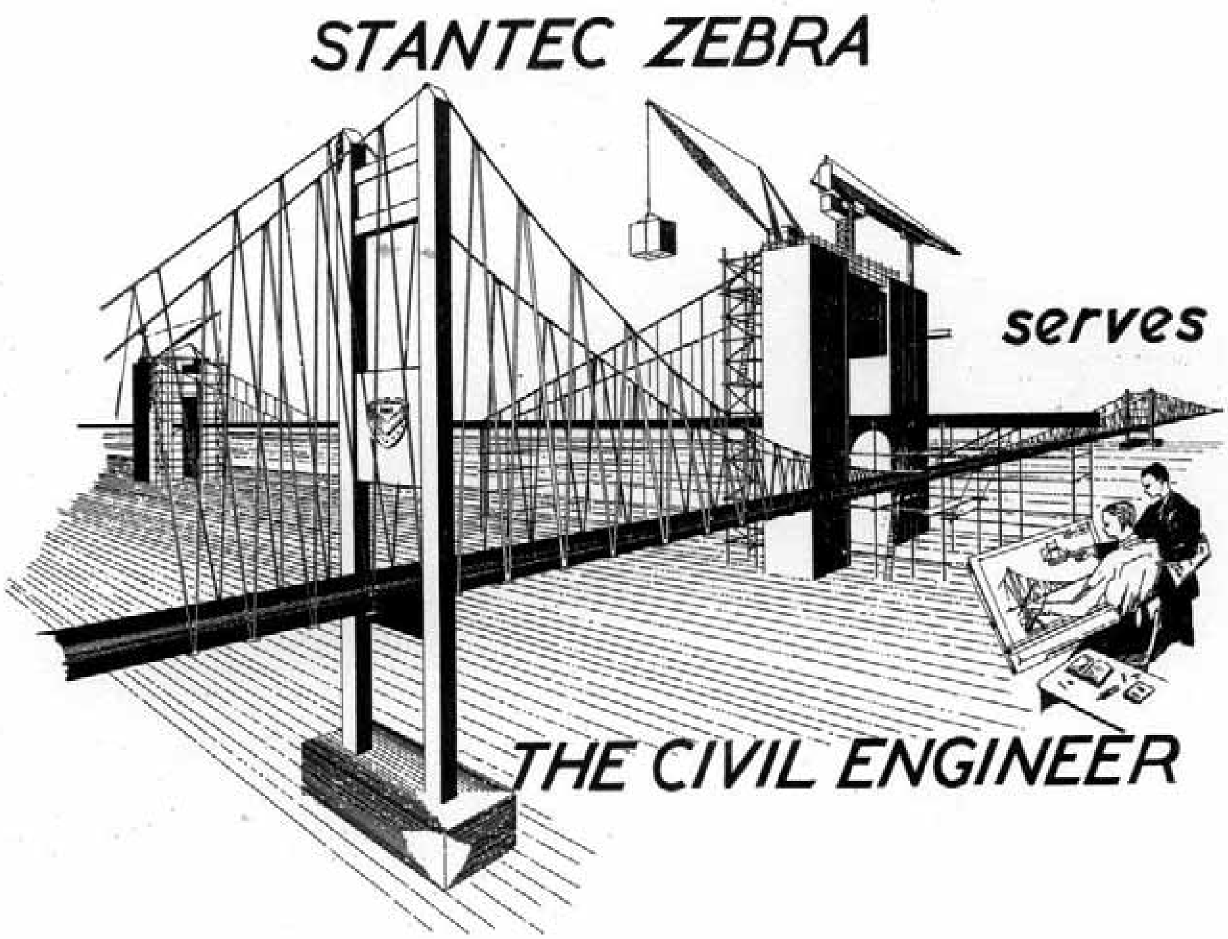
COMPUTER ANALYSIS OF STRUCTURES

BY

J. FERRY BORGES

Standard Telephones and Cables Limited

A. Cadete



COMPUTER ANALYSIS OF STRUCTURES

BY

J. FERRY BORGES

STANDARD TELEPHONES AND CABLES LIMITED
INFORMATION PROCESSING DIVISION
CORPORATION ROAD
NEWPORT, MON.

MINISTERIO DAS OBRAS PUBLICAS

LABORATORIO NACIONAL DE ENGENHARIA CIVIL

II SERVICO

SECCAO DE ESTRUTURAS

Proc. 732-II

COMPUTER ANALYSIS OF STRUCTURES

by

J. Ferry Borges

Research Engineer, Head, Structural Studies Section,
Laboratorio Nacional de Engenharia Civil, Lisbon,
Portugal.

Paper presented at the

II CONFERENCE ON ELECTRONIC COMPUTATION

Pittsburgh, September, 1960

Lisbon
June, 1960

COMPUTER ANALYSIS OF STRUCTURES

SYNOPSIS

A method based on the principle of transformation is presented, which unifies flexibility and stiffness methods.

Programs for the analysis of plane rectangular frames and plane frames in general, with bars of constant and variable section, are discussed and exemplified.

Structural analysis of normal and special types of suspension bridges, based on separate study of cable and beam, is also presented and exemplified. Results are compared with those obtained by model tests.

Reference is made to the computer used and to some special features of the programs.

1 - INTRODUCTION

The method of structural analysis based on the notion of transformation, besides being quite general, makes possible a very useful unification of concepts. This method can be expressed in a particularly simple form in a matrix language being consequently suitable for computers.

The method consists in transforming a structure, the behaviour of which is unknown, into another of known behaviour by changing its internal or external connections. Two main types of transformations are considered: cuts and fixations.

In the case of structures made up of bars, forces and moments at each cross section of a bar have six components. The transformation can deal in separate with each of these components or with the correspondent displacements. Thus 12 different elementary transformations can be introduced in the cross section of a bar: 6 corresponding to cuts, each of them making one of the force or moment components to vanish, and 6 corresponding to fixations each making a displacement component to vanish. Obviously other types of transformations could be considered among which e. g. elastic restraints. The preceding transformation types can be extended without difficulty to structures of other types (1).

When the transformation of the structure consists exclusively in the application of cuts, the method is usually called flexibility method; if in the introduction of fixations, stiffness method. The duality of these methods has been remarked and employed with success (see e. g. Argyris (2)). It is noteworthy that both types of transformations can be simultaneously applied a general method being consequently obtained, the transformation method.

This method consists, first, in transforming the structure whose behaviour is unknown into another of known behaviour, forces or displacements being then applied at the points affected by the transformation so as to reconstitute the original structure. This implies to know the behaviour of the transformed structure under both the action of the system of forces or displacements acting on the transformation points (cuts or fixations). The latter behaviour can be expressed by means of a matrix, the elements of which represent the effects of unit transformations. This matrix is called the transformation matrix, T.

The effect, at the transformation points, of forces applied at the transformed structure can be expressed by means of a vector, T_o. The components of this vector will be displacements, if the transformation consists in cuts, and forces if the transformation consists in fixations.

The forces and displacements, X, required to reconstitute the original structure have to comply with the condition $TX = - T_o$, where from $X = - T^{-1} T_o$. If the values of X are thus determined, the behaviour of the original structure becomes known.

Let us analyse, more in detail, the matrix T. This matrix can be represented in outline as indicated in fig. 1. Matrices $\underline{F}$ and $\underline{S}$ are those usually considered in, respectively, the flexibility and the stiffness methods. Two new matrices, $\underline{C}$ and $\underline{E}$ are now required, both depending on the geometric and elastic features of the structure.

As shown below, other transformations are useful besides those required to transform a statically indeterminate into a statically determinate structure. The behaviour of structures with widely different shapes is at present tabulated it being easy to resort to this knowledge. In the particular case of framed structures it is possible, by means of the well-known method of joint fixation, to obtain a system of fixed-end bars the behaviour of which can be expressed by simple formulas.

The advantage of adopting flexibility or stiffness methods can be assessed by means of the concepts of static and kinematic indeterminating (3). The adoption of the referred general method can also be advisable in most cases. Let us consider for instance the structure represented in fig. 2, a). By means of the transformations shown in b), a structure can be obtained, the analysis of which is simple. The adopted transformation consists in locking two joints and releasing a hanger. Fig. 3 shows how a statically determined structure can be studied by transformation into a statically indeterminate structure, what is achieved by locking its joints. This can be useful if, for instance the behaviour of the built-in arch is previously known.

2 - STUDY OF PLANE RECTANGULAR FRAMED STRUCTURES

The stiffness method is particularly suited to the study of plane rectangular frames since, rotations of the joints and possible translations normal to the axes of the bars being locked, the structure becomes a system of fixed-end bars, the behaviour of which is well-known. The number of fixations is thus equal to the number of joints that can rotate plus the number of possible translations.

The values of the bending moments and shearing forces at the fixed-end bars, when a unit rotation is applied at each end or a unit translation at one end with respect to the other, are shown in fig. 4. For a general bar, ij, four parameters, Kij, Kji, K'ij and Lij suffice to define these moments and the corresponding shearing forces.

The same figure shows how to determine these parameters for a bar with a variable cross-section in function of its geometric and elastic characteristics. In the particular case of a bar with a constant moment of inertia, as $K_{ij} = K_{ji} = 2\,K'_{ij} = \frac{4EI}{L}$, the problem grows simpler and the behaviour of the bar can be defined by means exclusively of K_{ij} and L_{ij}.

In order to obtain the stiffness matrix S, it is advisable to consider in separate, three sub-matrixes A, B and C, fig. 5, and to analyse the law of their formation. This is synthetically indicated in the same figure. Notice that besides the correspondence between the indices defining the bar and the constants K and L, it is necessary to establish an auxiliary correspondence between the subscripts of the translations and the subscripts of the joints directly affected by the translation, separating the joints placed above (right) and below (left) the bars undergoing a given horizontal (vertical) translation from those placed on the bars which undergo that translation themselves.

In order to facilitate, as far as possible the indications to be supplied by whoever requests the computation, it will suffice to indicate the geometric characteristics of the structure, fixed-end moments, statically-determinate reactions and concentrated forces acting in the direction of possible translations, by filling out Table A of fig. 6, which concerns bars with a constant cross-section. An additional Table B, in which the joints affected by each translation are indicated, is filled out at the computing centre.

Feeding the indications contained in both tables into the computer, it supplies matrix S and the vector S_o corresponding to the forces applied.

Rotations and translations are very easily determined by constructing the product - $S^{-1}\,S_o$. Bending moments and shearing forces are automatically obtained from the rotations and translations by means of simple linear expressions.

Fig. 7 shows how the computer presents the results by indicating successively the values of rotations, translations, bending moments and shearing forces.

program was presented by Livesley (4), in which only external forces are considered at the joints but in which the elastic stability is taken into account.

Notice that this program being more general than the one presented in 2, the inversion of a higher rank matrix is required. In the computer available, matrix inversion is directly possible only up to rank 62, what limits to 62 the number of fixations and consequently to 20 the number of joints.

4 - STUDY OF SUSPENSION BRIDGES

Classical suspension bridges with vertical hangers can be easily analysed by the deformability method. By cutting the hangers, the structure is transformed into two separate structures, a cable and a beam.

It is easy to obtain the expressions by means of which the displacements of a cable acted upon by concentrated forces can be computed (5). The only difficulty lies in the fact that these expressions are not linear i.e. the displacements obtained from them are not proportional to the applied forces. These expressions nevertheless can be rendered linear by determining the displacements due to a given unit force and assuming that the remaining values are proportional to the displacements thus obtained.

The unit force should be so selected that the total increases of the thrust obtained and the thrust due the forces to be considered are not very different. Notice that if a constant force is adopted, the matrix obtained is asymmetric as the reciprocal principle is not obeyed. Another possible method, would be to change the value of the forces considered so as to obtain constant thrust increases.

Terms taking into account the effect of the horizontal deformability of the towers and the elastic elongation of the cables can be easily introduced into the expressions giving the displacements of the cable.

Linear formulas can be easily deduced for determining the displacements of the beam, whether this is continuous or not.

Expressions by means of which the displacements of a cable or of a continuous beam with a constant moment of inertia and a central span three times the lateral spans can be computed, are indicated in fig. 9 and 10. These formulas can be easily calculated in the computer, yielding matrices $\underline{C}$ and $\underline{B}$, defining the flexibility of both types of members.

The concentrated forces on hangers, $\underline{X}$, can be calculated by the expression:

$$\{X\} = - [B - C]^{-1} \times \{B\}$$

in which $\underline{B}$ is a vector defining the flexibility of the beam for a given position of the force.

From X, influence diagrams for bending moments are easily obtainable.

The preceding method was applied in the analytical design of a bridge with a central span of 1,200 m, which had also been studied in model (*). A comparison is presented in fig. 12 between two influence lines obtained by analytical and experimental methods. The analytical solution was obtained by inversion of a matrix of rank 35. The agreement between the results should be remarked and it must be noted that the computed and tested structures were not completely similar. If it were so a better agreement would have been obtained.

In order to investigate up to what extent a variation in the number of equations influences the accuracy of the results, a matrix of rank 17 was also formed by alternately striking out rows and columns. The results obtained by inversion of this matrix differed from the first by about 15 per cent, what shows the need, in structures of this type, to express the problem by means of a considerable number of equations.

Besides being extremely simple, this analytical design method for suspension bridges, has the great advantage, in comparison with the traditional method based on the integration of the differencial equation, that it can be easily applied to suspension bridges other than those with vertical hangers only, and also to bridges in which the cable is anchored at the central point of the truss or where inclined cables are introduced.

In order to study, e.g. the effects of anchoring the cable at the central point of the truss, it suffices to consider an additional cut by means of which freedom of relative displacements is made possible at this point. This amounts to considering another equation, the coefficients of which are the horizontal relative displacements of the middle point of the cable and of the truss under the action of unit forces applied at the hangers of the transformed structure. The independent terms corresponding to this equation are the horizontal displacements of the anchoring point when the forces act on the truss only. The expressions by means of which these displacements can be determined are easy to obtain.

A computation carried out assuming the cable to be anchored at the truss, showed that the forces at the hangers changed by less than 5 per cent.

This method is also suitable for studying other special types of suspension bridges such as that outlined in fig. 12 a). This type of bridge has only an upper cable, the stiffness truss being replaced by two prestressed cables. The hangers and other horizontal cables in the plane of the prestressed ones form a space tetrahedral mesh (* *).

The behaviour of this bridge under the action of vertical forces can be studied by means of cuts at the lower cables in each panel. The system is thus transformed into a suspended cable, with oblique hangers forming triangular meshes, fig. 12, b. Once the displacements at the lower ends of the hangers under the action of vertical and horizontal forces are determined, what is easily done from the behaviour of the cable, there is no difficulty in writing the equations which will solve the problem.

A study of the space behaviour of this structure under the action of forces with any direction leads to a band matrix of very high rank (several hundreds). The fact that, except for a few diagonals near the main diagonal, all the elements of this matrix are zero, leads us to believe that it will not be difficult to construct the inverse matrix. This problem is at present being studied.

(*) These model tests were carried out by Prof. Edgar Cardoso, Professor of Bridge Engineering at Institute Superior Tecnico, Lisbon.

(**) A bridge of this type was suggested by Prof. Edgar Cardoso for crossing the Tagus in Lisbon.

5 - CONCLUSIONS

Transformation method being a powerful tool of structural analysis and having a simple matrix expression it is particularly suited for computers.

The analysis presented of frames and suspension bridges shows how adequate this method is for a medium-sized computer.

The general use, by the designers, of powerful means of analytical and experimental analysis available in specialized institutions, is a great help for the progress of structural engineering.

ACKNOWLEDGMENTS

The author wishes to thank Mr. Silverio Martins da Silva, Civil Engineer, for his help in the preparation of the present paper. The computations were carried out at the Applied Mathematics Section, Dr. Madalena Quirino was in charge of the programmation for frame analysis and Dr. Maria Margarida Cruz co-operated in the suspension bridge computations.

BIBLIOGRAPHY

1 - Clough, R.W. - Structural analysis by means of a matrix algebra program, Conference on Electronic Computation, American Society of Civil Engineers, Kansas City, November, 1958.

2 - Argyris, J.H. - Die Matrizentheorie der Statik, Ingenieur Archiv, Drittes Heft, Berlin, 1957.

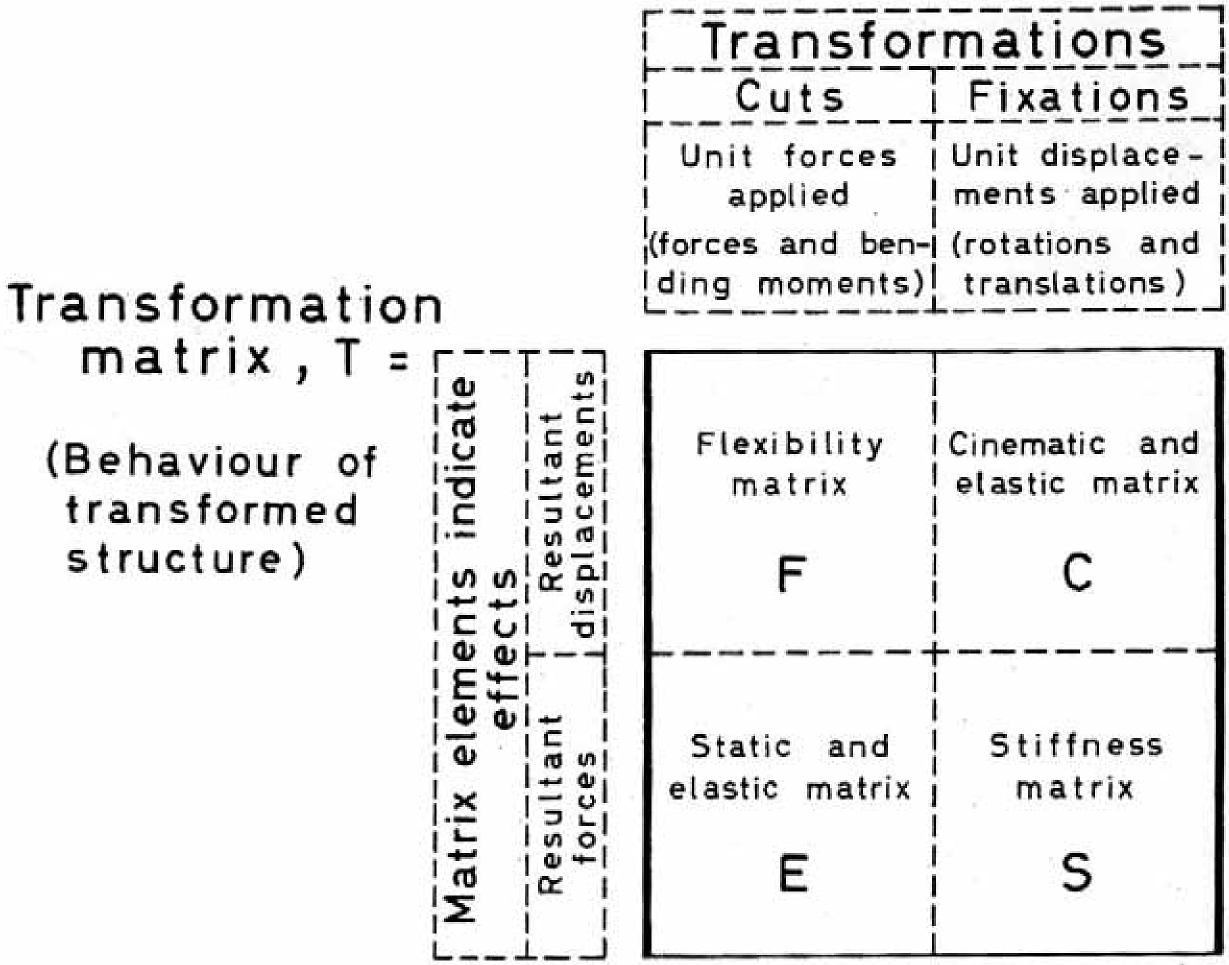

Transformations
Cuts
Fixations
Unit forces applied
(forces and bending moments)
Unit displacements applied
(rotations and translations)
Transformation matrix, T =
(Behaviour of transformed structure)
Matrix elements indicate effects
Resultant displacements
Resultant forces
Flexibility matrix
F
Cinematic and elastic matrix
C
Static and elastic matrix
E
Stiffness matrix
S

Fig. 1

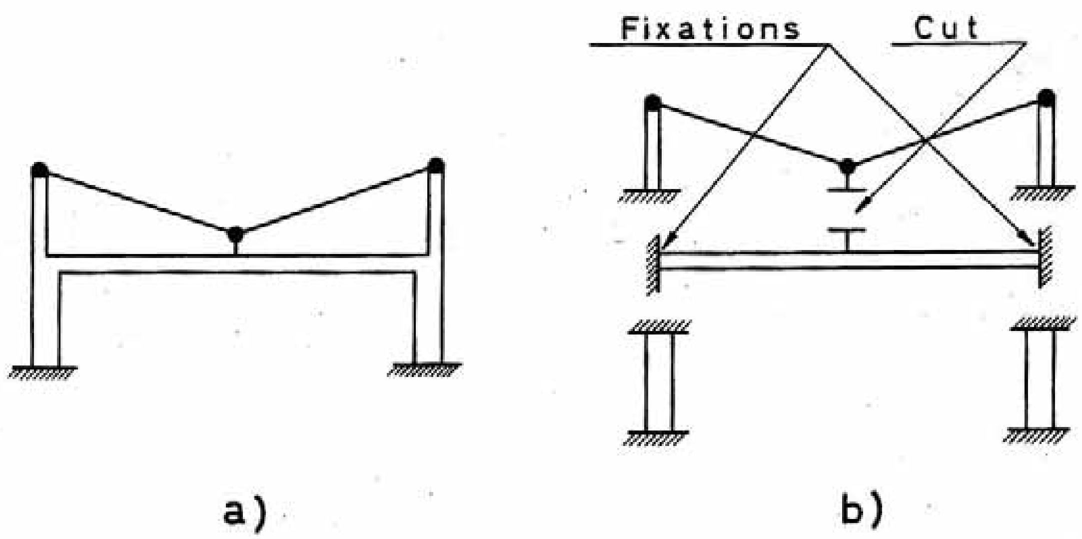

Fixations
Cut
a)
b)

Fig. 2

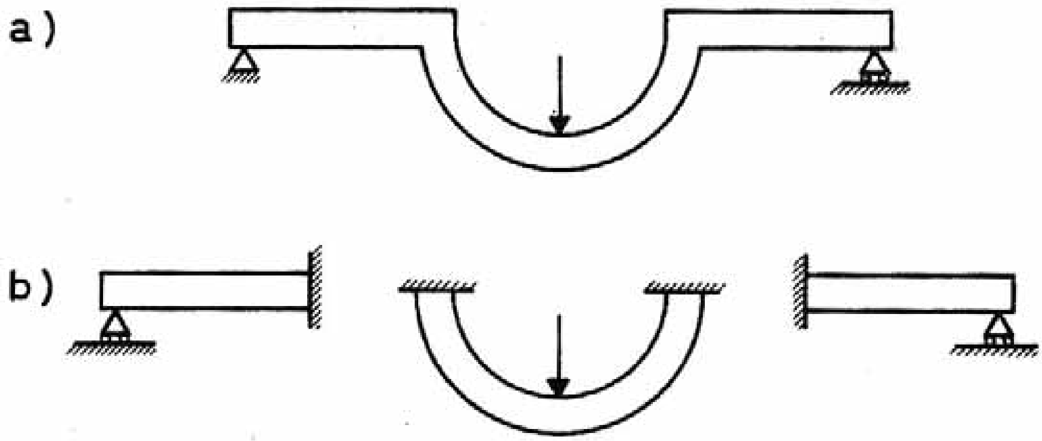
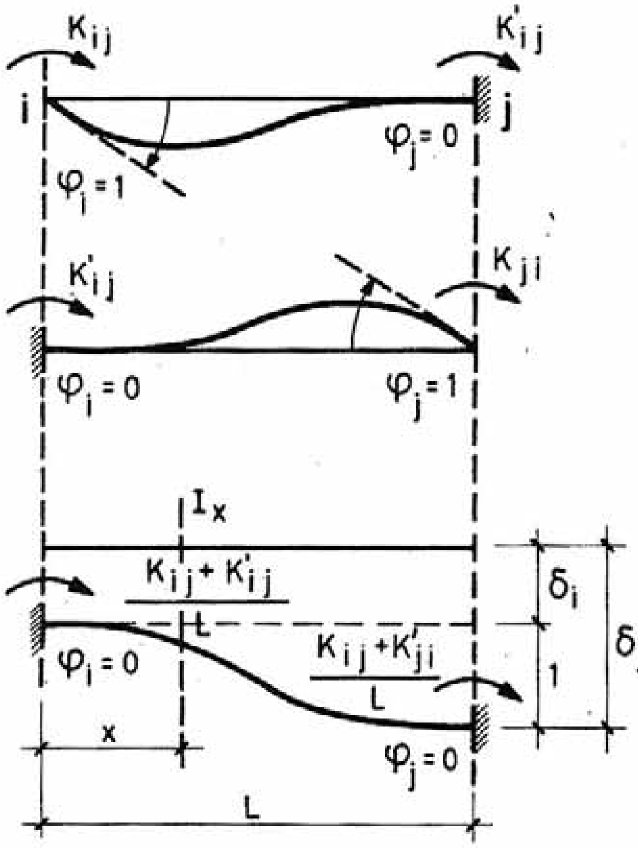


Fig. 3



I — Moment of inertia

E — Modulus of elasticity

K_{ij} — Stiffness factor

$K'_{ij} = \beta_{ij} K_{ij} = \beta_{ji} K_{ji}$

Constant inertia bar

$$K_{ij} = K_{ji} = \frac{4EI}{L}$$

$$\beta_{ij} = \beta_{ji} = 0.5$$

$$K'_{ij} = \frac{2EI}{L}$$

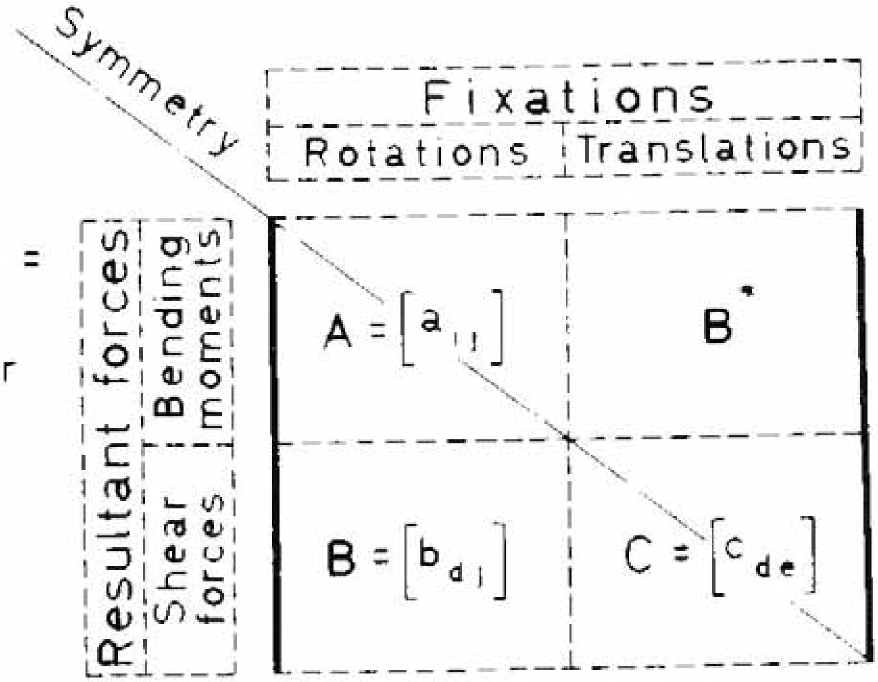
Variable inertia bar

$$\beta_{ij} = -1 + \frac{L \int_0^L \frac{x}{I_x}\, dx}{\int_0^L \frac{x^2}{I_x}\, dx} \qquad K_{ij} = \frac{E}{\int_0^L \frac{dx}{I_x} - \frac{1+\beta_{ij}}{L} \int_0^L \frac{x}{I_x}\, dx}$$

Fig. 4

Stiffness matrix, S =

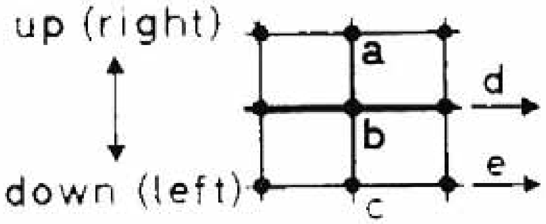
(Rectangular frame)



i , j – index corresponding to rotation

d , e – index corresponding to translation

a, b, c – points directly connected to, or on the bar that suffers displacement d



Correspondance d → a, b, c established on a table

$$a_{ii} = \sum_h K_{ih} + \sum_h K_{hi} \qquad a_{ij} = K'_{ij}$$

$$b_{dj} \rightarrow j = a \quad b_{da} = \frac{K_{ab} + K'_{ab}}{L_{ab}} \qquad j = c \quad b_{dc} = -\frac{K_{cb} + K'_{bc}}{L_{bc}}$$

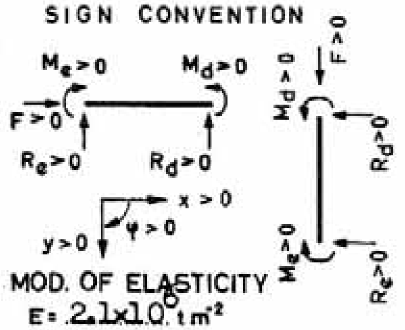
$$j = b \quad b_{db} = \frac{K_{ba} + K'_{ab}}{L_{ab}} - \frac{K_{bc} + K'_{bc}}{L_{bc}}$$

$$c_{dd} = \sum \frac{K_{ab} + K_{ba} + 2K'_{ab}}{L^2_{ab}} + \sum \frac{K_{bc} + K_{cb} + 2K'_{bc}}{L^2_{bc}}$$

$$c_{de} = -\sum \frac{K_{bc} + K_{cb} + 2K'_{bc}}{L^2_{bc}}$$

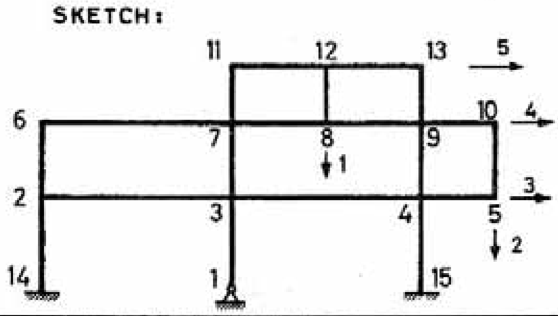
Fig. 5

SIGN CONVENTION



MOD. OF ELASTICITY $E = 2.1 \times 10^6\ t\,m^{-2}$

SKETCH:



BAR Left (down) end	BAR Rigth (up) end	GEOMETRIC CHARACTERISTICS Length L (m)	GEOMETRIC CHARACTERISTICS Moment of inertia $I \times 10^4 (m^4)$	FIXED-END MOMENTS Left M_e (tm)	FIXED-END MOMENTS Rigth M_d (tm)	ISOSTATIC REACTIONS Left R_e(t)	ISOSTATIC REACTIONS Rigth R_d(t)	DISP. f	External force applied at the displaced bar F (t)
14	2	5.0	21:3	0	0	0	0	1	0.0
2	3	10.0	333.3	-40.0	-40.0	24.0	24.0	2	5.0
1	3	5.0	72.0	0	0	0	0	3	15.0
3	4	10.0	333.0	-40.0	-40.0	24.0	24.0	4	0.0
15	4	5.0	72.0	0	0	0	0	5	0.0
4	5	4.0	170.7	- 6.4	- 6.4	9.6	9.6		
2	6	4.0	9.0	0	0	0	0		
6	7	10.0	72.0	-23.2	-32.8	14.0	20.0		
3	7	4.0	21.3	0	0	0	0		
7	8	5.0	41.7	-10.0	-10.0	12.0	12.0		
8	9	5.0	41.7	-10.0	-10.0	12.0	12.0		
4	9	4.0	21.3	0	0	0	0		
9	10	4.0	170.7	- 3.2	- 3.2	4.8	4.8		
5	10	4.0	9.0	0	0	0	0		
7	11	3.0	21.3	0	0	0	0		
11	12	5.0	333.3	- 5.0	- 5.0	6.0	6.0		
8	12	3.0	9.0	0	0	0	0		
12	13	5.0	333.3	- 5.0	- 5.0	6.0	6.0		
9	13	3.0	21.3	0	0	0	0		

TABLE A

DISP.		ENDS AFFECTED BY THE DISPLACEMENT				
1	a	9	13			
	b	8	12			
	c	7	11			
2	a					
	b	5	10			
	c	4	9			
3	a	6	7	9	10	
	b	2	3	4	5	
	c	14	1	15		
4	a		11	12	13	
	b	6	7	8	9	10
	c	2	3		4	5
5	a					
	b	11	12	13		
	c	7	8	9		

TABLE B

Fig. 6

ROT. (MILLIMETERS PER METER)

1	=+2.6508	2	=+1.5033	3	=-0.1360	4	=+0.8131
5	=+2.1133	6	=+3.3499	7	=-0.8212	8	=-0.0268
9	=+0.9799	10	=+2.1190	11	=+1.7650	12	=+0.0013
13	=-1.7650						

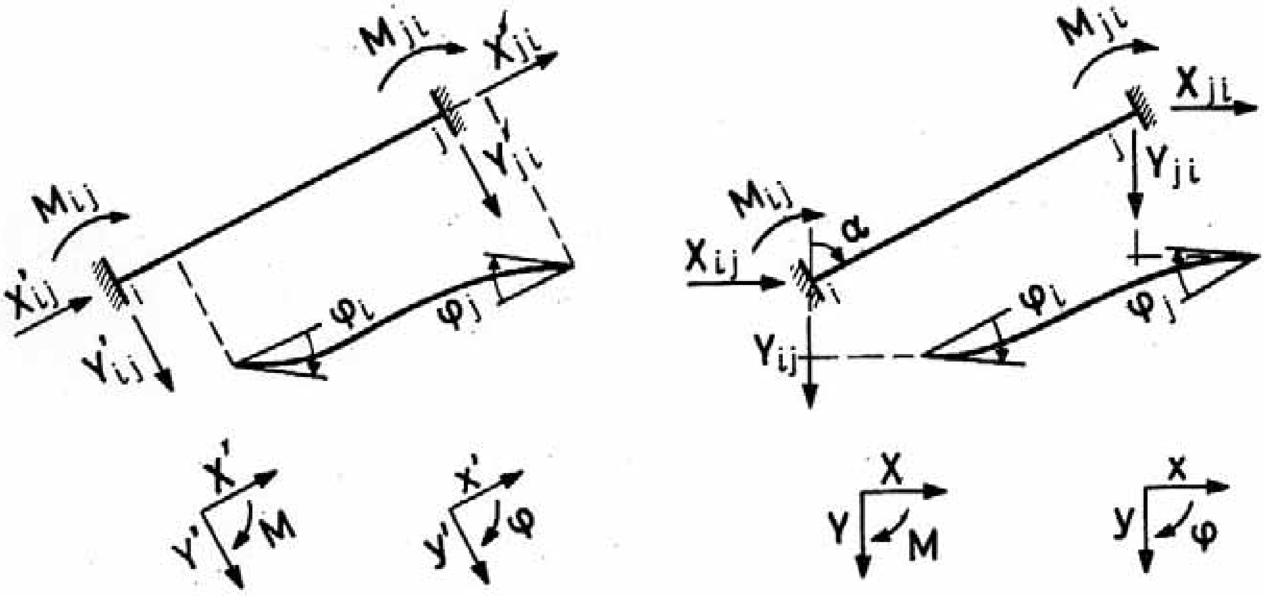
TRANSL. (MILLIMETERS)

1	=+ 6.05	2	=+ 7.47	3	=+ 8.61	4	=+ 11.90
5	=+ 11.99						

BAR		INITIAL MOMENTS (METRIC TONS.METER) LEFT	RIGHT	FINAL MOMENTS (METRIC TONS METER) LEFT	RIGHT
14	2	+ 0.000	+ 0.000	- 6.553	+ 3.863
2	3	-40.000	-40.000	+ 0.185	-57.237
1	3	+ 0.000	+ 0.000	- 0.000	+16.855
3	4	-40.000	-40.000	-32.425	-60.861
15	4	+ 0.000	+ 0.000	-26.324	+21.406
4	5	- 6.400	- 6.400	-39.771	+ 3.667
2	6	+ 0.000	+ 0.000	+ 3.678	- 5.423
6	7	-23.200	-32.800	- 5.423	-37.964
3	7	+ 0.000	+ 0.000	- 7.957	+ 9.490
7	8	-10.000	-10.000	-28.567	+ 5.784
8	9	-10.000	-10.000	+ 5.965	-29.491
4	9	+ 0.000	+ 0.000	+ 0.316	- 0.689
9	10	- 3.200	- 3.200	-30.489	+ 3.673
5	10	+ 0.000	+ 0.000	+ 3.667	- 3.673
7	11	+ 0.000	+ 0.000	+ 0.092	- 7.804
11	12	- 5.000	- 5.000	- 7.804	+47.182
8	12	+ 0.000	+ 0.000	- 0.181	+ 0.146
12	13	- 5.000	- 5.000	+47.328	- 7.877
9	13	+ 0.000	+ 0.000	+ 0.308	+ 7.877

BAR		ISOSTATIC REACTIONS (METRIC TONS) LEFT	RIGHT	FINAL SHEARING FORCES (METRIC TONS) LEFT	RIGHT
14	2	+ 0.000	+ 0.000	+ 2.083	+ 2.083
2	3	+ 24.000	+ 24.000	+ 18.258	- 29.742
1	3	+ 0.000	+ 0.000	+ 3.371	+ 3.371
3	4	+ 24.000	+ 24.000	+ 21.156	- 26.844
15	4	+ 0.000	+ 0.000	+ 9.546	+ 9.546
4	5	+ 9.600	+ 9.600	+ 20.460	+ 1.260
2	6	+ 0.000	+ 0.000	- 2.275	- 2.275
6	7	+ 14.000	+ 20.000	+ 10.746	- 23.254
3	7	+ 0.000	+ 0.000	+ 4.362	+ 4.362
7	8	+ 12.000	+ 12.000	+ 18.870	- 5.130
8	9	+ 12.000	+ 12.000	+ 4.909	- 19.091
4	9	+ 0.000	+ 0.000	- 0.251	- 0.251
9	10	+ 4.800	+ 4.800	+ 13.340	+ 3.740
5	10	+ 0.000	+ 0.000	- 1.835	- 1.835
7	11	+ 0.000	+ 0.000	- 2.632	- 2.632
11	12	+ 6.000	+ 6.000	+ 16.997	+ 4.997
8	12	+ 0.000	+ 0.000	+ 0.109	+ 0.109
12	13	+ 6.000	+ 6.000	- 5.041	- 17.041
9	13	+ 0.000	+ 0.000	+ 2.523	+ 2.523

Fig. 7



$$D'_i = \begin{bmatrix} x'_i \\ y'_i \\ \varphi'_i \end{bmatrix} \qquad V'_{ij} = \begin{bmatrix} X'_{ij} \\ Y'_{ij} \\ M'_{ij} \end{bmatrix} \qquad D_i = \begin{bmatrix} x_i \\ y_i \\ \varphi_i \end{bmatrix} \qquad V_{ij} = \begin{bmatrix} X_{ij} \\ Y_{ij} \\ M_{ij} \end{bmatrix}$$

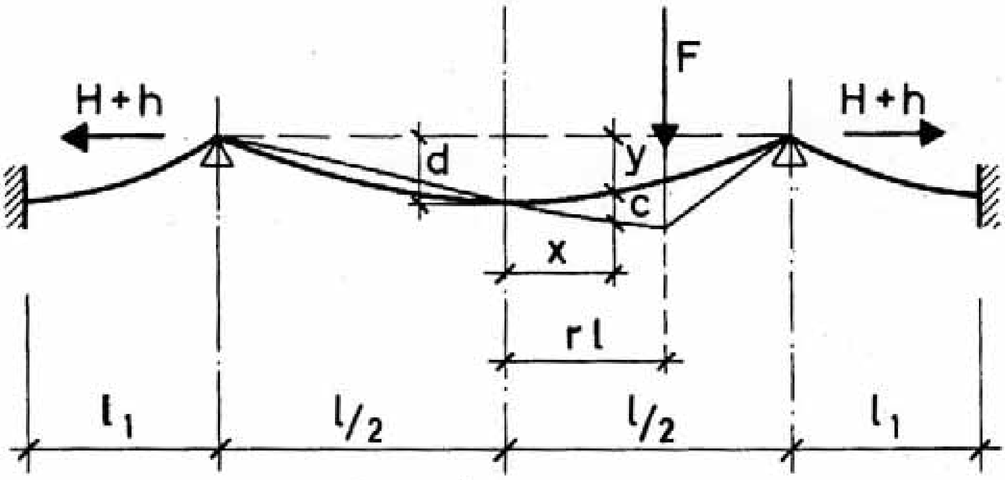
$$D'_i = G\,D_i \qquad V'_{ij} = G\,V_{ij} \qquad G = \begin{bmatrix} \operatorname{sen}\alpha & -\cos\alpha & 0 \\ \cos\alpha & \operatorname{sen}\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S'_{ii} = \begin{bmatrix} K'' & 0 & 0 \\ 0 & \dfrac{K_{ij} + K_{ji} + 2K'_{ij}}{L^2_{ij}} & \dfrac{K_{ij} + K'_{ij}}{L_{ij}} \\ 0 & \dfrac{K_{ij} + K'_{ij}}{L_{ij}} & K_{ij} \end{bmatrix} \qquad K'' = \dfrac{E}{\int_0^L \dfrac{dx}{A_x}}$$

$$S'_{ij} = \begin{bmatrix} -K'' & 0 & 0 \\ 0 & -\dfrac{K_{ij} + K_{ji} + 2K'_{ij}}{L^2_{ij}} & \dfrac{K_{ij} + K'_{ij}}{L_{ij}} \\ 0 & \dfrac{K_{ij} + K'_{ij}}{L_{ij}} & K'_{ij} \end{bmatrix}$$

$$S_{ii} = G^{-1} S'_{ii} G \qquad S_{ij} = G^{-1} S'_{ij} G$$

Fig. 8



$$y = \frac{w}{2H}x^2 \qquad H = \frac{wl}{8d}$$

w - weight per unit length

$$h = \frac{3H}{2}\ \frac{F}{wl}(1-4r^2)$$

$$c_o = d - \frac{wl^2}{8(H+h)}\left[1+\frac{F}{wl}(1-2r)\right]^2$$

$$-\frac{l}{2} \leq x \leq rl$$

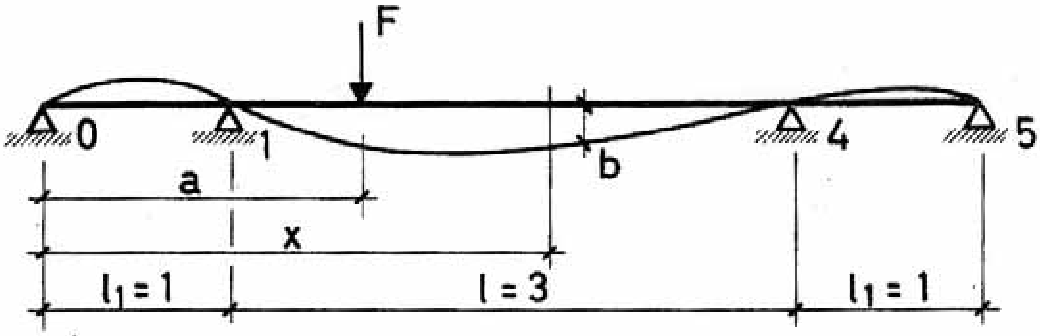
$$c = c_o + \frac{w}{2(H+h)}\left[x - \frac{F}{w}(0{,}5-r)^2\right]^2 - \frac{w}{2H}x^2$$

$$rl \leq x \leq \frac{l}{2}$$

$$c = c_o + \frac{w}{2(H+h)}\left[x - \frac{F}{w}(0{,}5+r)^2\right]^2 -$$

$$- \frac{F}{H+h}\ rl\left(1+\frac{F}{wl}\right) - \frac{w}{2H}x^2$$

Fig. 9



E - Modulus of elasticity I - Moment of inertia

1 – Load on left span , $0 \leq a \leq 1$

$0 \leq x \leq a$

$$b = \frac{F}{6EI} \cdot \left[x(1-a)(2a-a^2-x^2) - \frac{8a(1-a^2)}{55} \cdot (x-x^3) \right]$$

$a \leq x \leq 1$

$$b = \frac{F}{6EI} \cdot \left\{ \left[x(1-a)(2a-a^2-x^2) + (x-a)^3 \right] - \frac{8a(1-a^2)}{55} \cdot (x-x^3) \right\}$$

$1 \leq x \leq 4$

$$b = \frac{F}{6EI} \cdot \frac{a(1-a^2)}{55} \left\{ -8\left[\frac{(x-1)^3}{3} - 3(x-1)^2 + 6(x-1) \right] + 3\left[3(x-1) - \frac{(x-1)^3}{3} \right] \right\}$$

$4 \leq x \leq 5$

$$b = \frac{F}{6EI} \cdot \frac{3a(1-a^2)}{55} \cdot \left[(x-4) - (x-4)^3 \right]$$

2 – Load on central span , $0 \leq a \leq 4$

$0 \leq x \leq 1$

$$b = -\frac{F}{6EI} \cdot \frac{8(a-1)(4-a)(7-a) - 3(a-1)\left[9-(a-1)^2\right]}{165} \cdot (x-x^3)$$

$1 \leq x \leq a$

$$\begin{aligned} b = \frac{F}{6EI} \Bigg\{ & \frac{x-1}{3}(4-a) \cdot \left[6(a-1) - (a-1)^2 - (x-1)^2 \right] - \\ & - \left[\frac{(x-1)^3}{3} - 3(x-1)^2 + 6(x-1) \right] \cdot \frac{8(a-1)(4-a)(7-a) - 3(a-1)\left[9-(a-1)^2\right]}{165} - \\ & - \left[3(x-1) - \frac{(x-1)^3}{3} \right] \cdot \frac{8(a-1)\left[9-(a-1)^2\right] - 3(a-1)(4-a)(7-a)}{165} \Bigg\} \end{aligned}$$

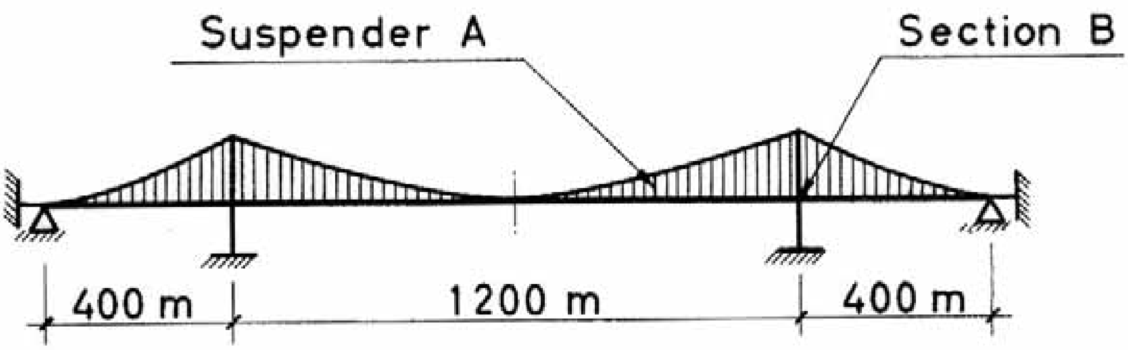
$a \leq x \leq 4$

$$\begin{aligned} b = \frac{F}{6EI} & \left\{ \frac{(x-1)}{3}(4-a) \cdot \left[6(a-1) - (a-1)^2 - (x-1)^2 \right] + (x-a)^3 \right\} - \\ & - \left[\frac{(x-1)^3}{3} - 3(x-1)^2 + 6(x-1) \right] \cdot \frac{8(a-1)(4-a)(7-a) - 3(a-1)\left[9-(a-1)^2\right]}{165} - \\ & - \left[3(x-1) - \frac{(x-1)^3}{3} \right] \cdot \frac{8(a-1)\left[9-(a-1)^2\right] - 3(a-1)(4-a)(7-a)}{165} \end{aligned}$$

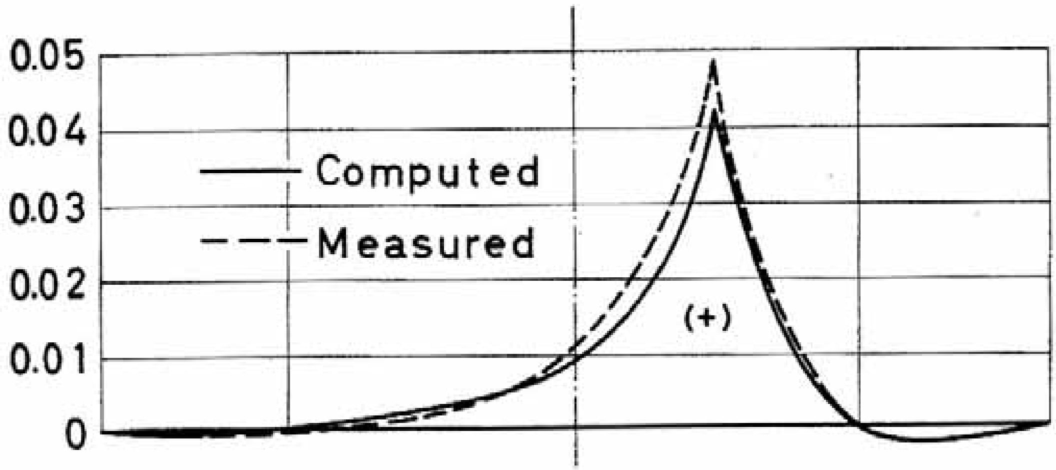
$4 \leq x \leq 5$

$$b = \frac{F}{6EI} \cdot \frac{8(a-1)\left[9-(a-1)^2\right] - 3(a-1)(4-a)(7-a)}{165} \left[(x-1)^3 - 3(x-1)^2 + 2(x-1) \right]$$

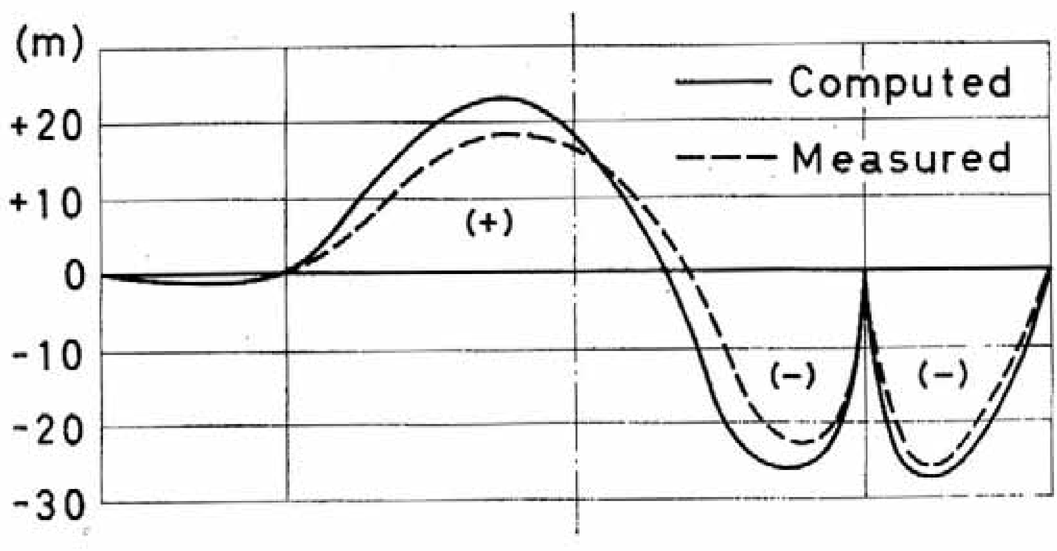
Fig. 10



(Distance between suspenders –25 m)

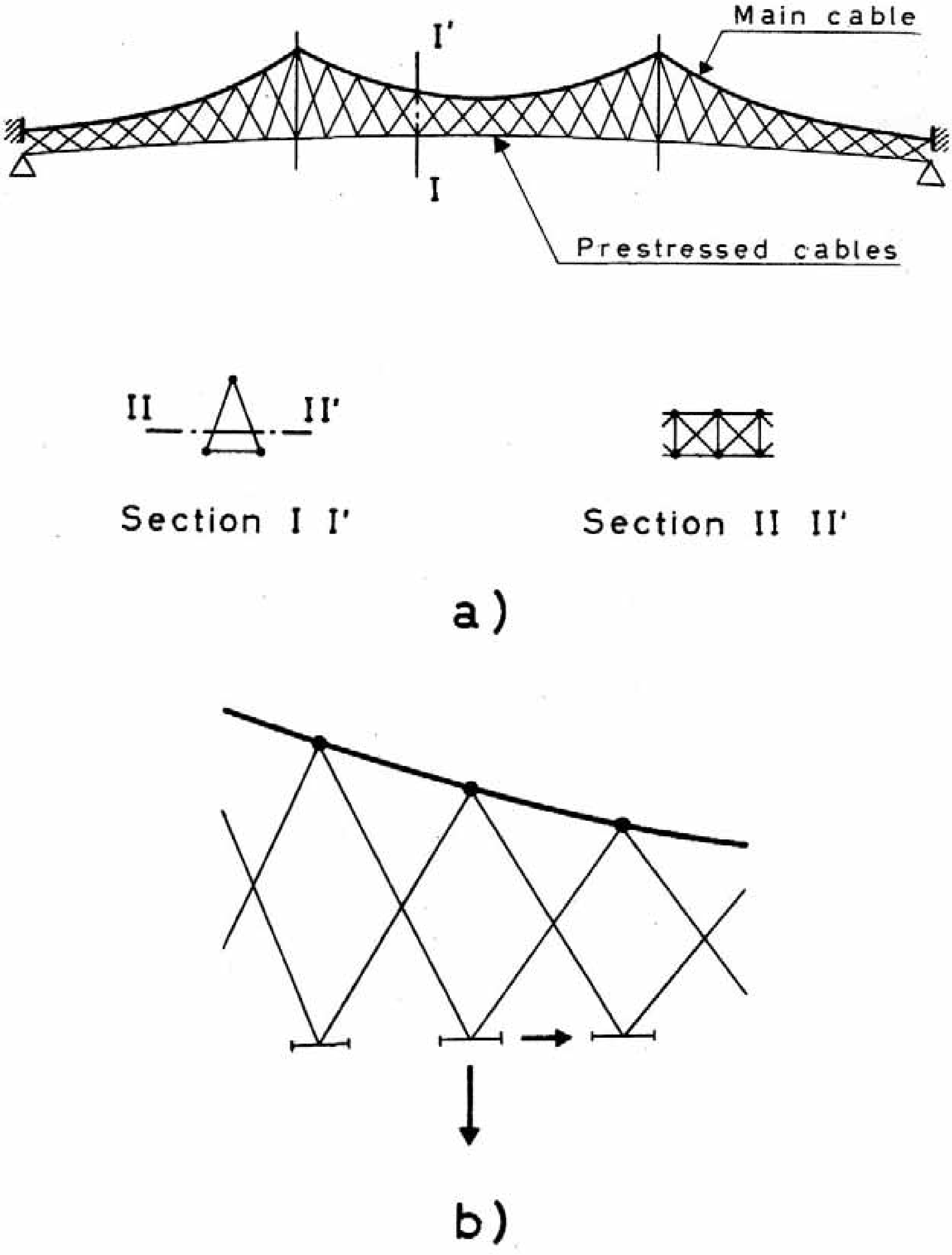


Influence line of normal force at suspender A



Influence line of bending moment at section B

Fig. 11



Main cable
I'
I
Prestressed cables
II
II'
Section I I'
Section II II'
a)
b)

Fig. 12